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Characteristics Of Implied Volatility In The S&P100 Options With European-Style Exercise

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Abstract

While the topic of volatility has been much further developed in the last three decades, I will try to revise the implied volatility and its characteristics based on the ten day data of Standard and Poor's 100 index European-style options (XEO). Having calculated the implied volatility of the collected options, further analyses are to study the impact of two important factors of time to maturity and moneyness on the value of implied volatility.

1. INTRODUCTION

The measure of Volatility estimates uncertainty in the changes of one or more financial instruments focusing on the period ranging from the moment of estimation/calculation until its maturity. It is can also be described as the difference between the expected change in the option and the used risk-free rate. On the other hand, it can be taken into consideration as a measure of risk of an investment, as it tells the fluctuation level of the expected return of a specific security or portfolio.

The research of volatility in the financial field was most seriously started by Bachelier (1900) who used Arithmetic Brownin Motion for pricing options in his PhD thesis, and was the first to use the Brownian motion in finance. Bachelier assumed the stock prices to follow a normal distribution. Among the weaknesses in his method is that the option price could be negative. Further developments of the Bachelier's work were done by Kendall (1953), who suggested randomness of the movement of market shares (likelihood equality of price decrease and increase), Osborne(1959), and Samuelson (1965), who replaced the Arithmetic Brownian Motion with the Geometric Brownian Motion, and assumed it on return rates (used as discount rates). Sprenkle (1961) used the development of use of Geometric Brownian Motion, through assuming lognormal distribution of the stock price, while valuing the risk-free-rate to be zero. One of the main achievements of his research is removing the possibility of calculating a negative option price. Mandelbrot (1963) and Fama (1965) identified the similarity of price changes in one day and in one month for the same financial instrument, which covered the period of 1900-1960 data. Boness (1964) and Samuelson (1965), having added the expected-rate value, unsuccessfully tried to adjust the risk measure.

The formula was adjusted by Black, Scholes (1973) and Merton (1973), using a risk-free-rate and is known as Black-Scholes-Merton formula.

2. IMPLIED VOLATILITY

It is also the formula used to evaluate the implied volatility in the European style options. The Black–Scholes–Merton formula is:

$$c = SN(d_1) - Ke^{-rT}\mathrm{N}(\mathrm{d}_2) \qquad (2.1)$$

And

$$p = Ke^{-rT}\mathrm{N}(-\mathrm{d}_2) - SN(-d_1) \qquad (2.2)$$

Where

$$d_1 = \frac{\ln(S/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad (2.3)$$

$$d_2 = d_1 - \sigma\sqrt{T} \qquad (2.4)$$

N(d2) represents a probability that option will be exercised. It gets multiplied by the strike price that is discounted to its present time value, giving the expected value of the cost of exercising an option. According to Lars Tyge Nielsen (1993), N(d1) is the factor by which the actual present value of the stock exceeds the current stock price. Statistically, N(-d) is equivalent to 1-N(d).

Black (1975) states that the market prices of listed options tend to methodically differ from the prices calculated by the Black-Scholes formula. He claims that strongly out of the money options tend to be overpriced, while into-the-money options tend to be underpriced. He also adds that options having time-to-maturity below three months to tend to be overpriced.

At its beginning, some researchers like Letane and Rendleman (1976) and Beckers (1981) used the same Black-Scholes model in valuing volatility for American style options, although Beckers did some adjustments assuming possible earlier option exercise.

Further studies on implied volatility were done by Melino and Turnbul (1990), Day and Lewis(1992), who used the S&P100 OEX options data and compared their implied volatility with the GARCH and EGARCH, and by Lamoureux and Lastrapes (1993), who relied on separate stock options, both concluding the usefulness of the implied volatility in combination with time-series models. Lamoureux and Lastrapes (1993) also find the outcomes of implied volatility to undervalue the actual volatility. On the other hand Canina and Figlewski (1993), furthermore using the OEX data, conclude that historical volatility outperforms the implied volatility. On the positive side, Jorion (1995) concludes implied volatility doing better than historical, comparing it to the moving average and to the GARCH method.

Campbell *et al.* (1997), describe the volatility to be essential to financial economics. They add that without uncertainty the problems of financial economics would not differ from basic microeconomics.

This paper is based on the data sample of S&P100 european-style options ($XEO), collected online, for ten trading days of the month of April 2014.

The real market data included all the options, with varying times to maturity, and varying strike prices. The time to maturity values were calculated manually, while the value used for the risk free rate, supported by Galai (1978) as well as Pablo Fernandez, Alberto Ortiz and Isabel F. Acin (2015), was the ten year Treasury bill rate.

The calculation and graphing were done using the Mathematica software:

```
calls = Import["calls.csv"];

d1[v_, S_, K_, r_, t_] = (Log[S / K] + (r + v^2 / 2) t) / ((v) Sqrt[t]);
d2[v_, S_, K_, r_, t_] = d1[v, S, K, r, t] - v Sqrt[t];
n1[v_, S_, K_, r_, t_] = CDF[NormalDistribution[], d1[v, S, K, r, t]];
n2[v_, S_, K_, r_, t_] = CDF[NormalDistribution[], d2[v, S, K, r, t]];

imv = Table[0, {i, 1, 797}];

ParallelCombine[Do[{
   S = calls[[j, 1]],
   K = calls[[j, 3]],
   c = calls[[j, 4]],
   t = calls[[j, 5]] / 252,
   r = calls[[j, 6]],
   imv[[j]] = NMinimize[{(
         c - S n1[v, S, K, r, t]
          + K Exp[-r t] n2[v, S, K, r, t] )^2 , v > 0.}, {v}] }, {j, 1, 797}] ]
```

3. ANALYZING THE IMPLIED VOLATILITY

The outcome of the above code included some best approximation implied volatility values, but not the exact answer; such cases were removed from further research. As such, the following analysis part of the outcomes will show some of these, grouped by time to maturity, groups having very few if any options with a valid implied volatility values.

3.1 Analyzing Put options clustered by time to maturity

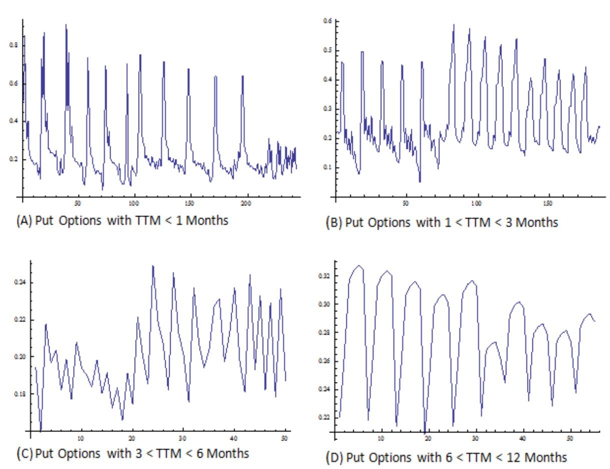

Figure 3.1 XEO Put options grouped by time to maturity

Analyzing the four groups in the Figure 3.1, the most stable IV values occur for the put options having Time to Maturity ranging from 3 to 6 months. The Standard Deviation of the I.V. for that group is the lowest of the four, at a value of 0.022. The second lowest is the group of Put options with the Time to Maturity ranging from 6 months to one year, having the standard deviation value of 0.034. The most varying IV is in the group of Put options having Time to Maturity less than one month.

While looking the change in time to maturity and its effect on a change in the mean value of its IV, order is slightly different. The same group of Time to Maturity ranging from 3 to 6 months has the lowest IV. On the other hand, the remaining three groups are in an ascending order by the Time to Maturity, starting by time to maturity less than one month, followed by time to maturity ranging from 1 to 3 month, followed by time to maturity ranging from 6 to 12 months.

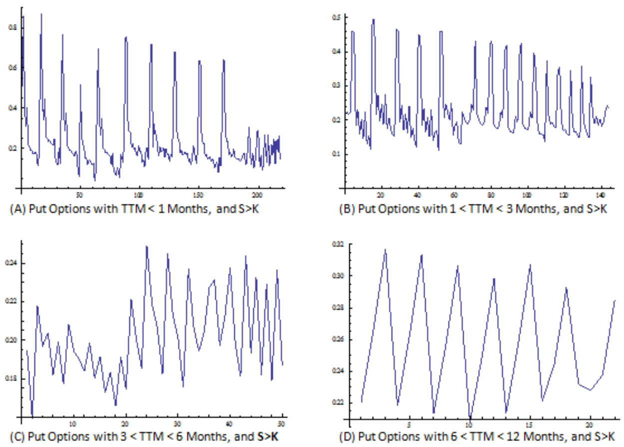

Figure 3.2 XEO Put options grouped by time to maturity, with S>K

Having divided the put options into two groups, as can be seen in Figure 3.2 that for group where Spot price is greater than the Strike price, the implied volatility standard deviation value is still the lowest for options with time to maturity ranging from 3 to 6 month, followed by time to maturity exceeding 6 month, then time to maturity ranging from 1 to 3 month, and highest with time to maturity below one month.

The order of IV mean value has slightly changed from all the puts, still minimal value holds for puts with time to maturity between 3 and 6 months, followed by puts with time to maturity between 1 and 3 months, then puts with time to maturity less than 1 month, and the maximal for puts with time to maturity more than 6 months.

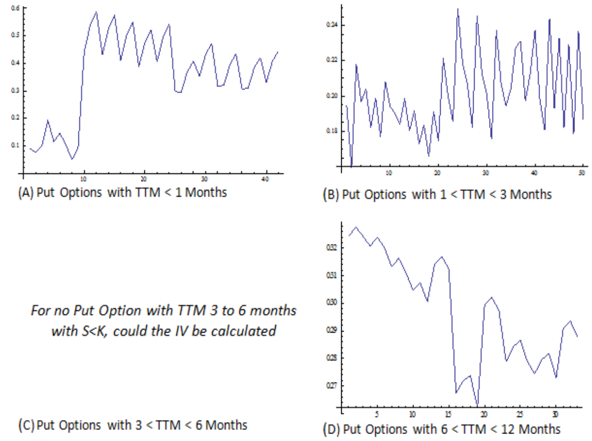

Figure 3.3 XEO Put options grouped by time to maturity, with S<K

The puts having less than 1 month time to maturity, and those with time to maturity ranging between 1 and 3 month have greater standard deviation with S<K than the same do with S>K. The opposite is true for puts with time to maturity greater than 6 months which turn to have the minimal standard deviation of the implied volatility compared to all the previously mentioned puts groups.

It should be noted that not a single put option having S<K and time to maturity between 3 and 6 month had a valid Black-Scholes-Merton formula calculated implied volatility. On the other hand for the remaining three groups, the mean value of the implied volatility with S<K is greater than the corresponding group with S<K.

3.2 Analyzing Call options clustered by time to maturity

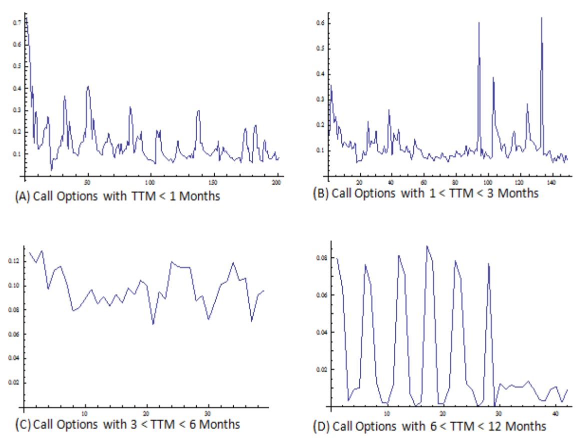

Figure 3.4 XEO Call options grouped by time to maturity

The Standard Deviation of the Implied Volatility for Call Options, shown in Figure 3.4, has similar characteristics as it does for Puts. Its value generally decreases as the Time to Maturity increases, except for the for the Options with Time to Maturity ranging between 3 and 6 months again having the lowest standard deviation value among the four groups.

On the other hand, looking at the movement of the mean value of the Implied Volatility, of each of these groups, as the time to maturity increases, it moves in opposite direction. So, the Implied Volatility decreases as time to maturity increases with no exceptions. As the groups represent times to maturity of one month, one to three months, three to six months, and six to twelve months, the implied volatility mean values for these groups are 0.149, 0.119, 0.0982, and 0.025 respectively.

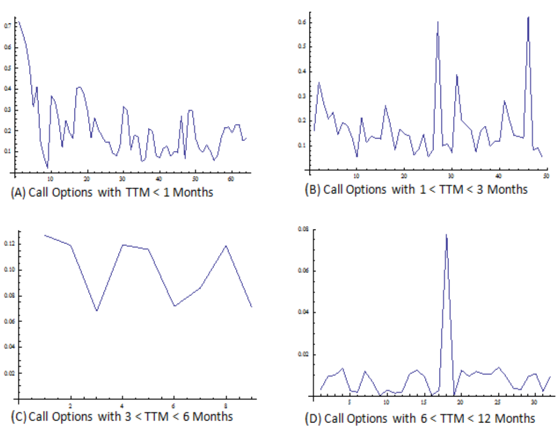

Figure 3.5 XEO Call options grouped by time to maturity, with S>K

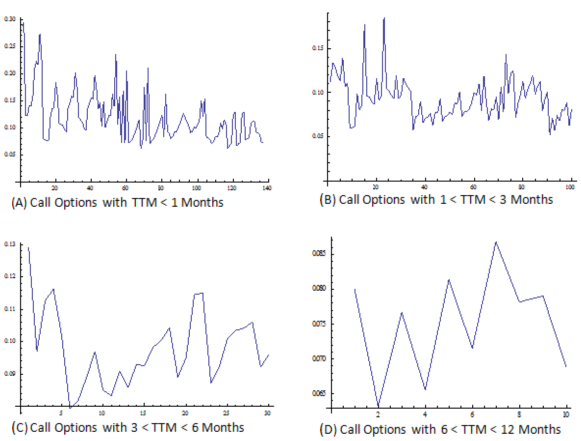

Figure 3.6 XEO Call options grouped by time to maturity, with S<K

Call options having the Spot price greater than the Strike price are being shown in Figure 3.5. Looking at the four groups in Figure 3.5, which are grouped based on the options' time to maturity, the mean values of implied volatility is decreasing as options' time to maturity increases. Similarly, as the time to maturity increases as such does the standard deviation of implied volatility decrease, accordingly.

Needed to mention again is that there were market options with time to maturity more than six months, and there were more options with time to maturity between three and six months, but they did not provide properly fitting implied volatility values, and thereby were removed from the data to be analyzed. As such, some groups can be considered to be inconclusive due to very limited amount of data.

Analyzing the order of the mean values of groups' implied volatility, for the call options having spot price less than strike price (Figure 3.6), it cannot be correlated to the changes in time to maturity. The maximum mean implied volatility value occurs with options having time to maturity less than one month, followed by time to maturity ranging between three and six months, followed by one to three months, and the lowest being with time to maturity exceeding six month.

As for the spread of the volatility values, it is very much related to the time to maturity, just in reverse order, as the time increases the lower the standard deviation of the implied volatility becomes.

4. Analyzing the Implied Volatility values, clustered by Moneyness

Calculating the implied volatility, for the options of same stock with the same time to maturity under different strike prices, would give such values of implied volatility with graph in form of a smile. MacBeth and Merville (1979) calculated implied volatility of European-style options using BSM of 6 different options for 1976, and found how it overestimates the out of money (strike price greater than spot price) options and underestimates the in-the-money (strike price less than spot price) options. Moneyness, representing the ratio of spot price versus strike price, is also known for the volatility smiley Beckers (1980).

In this part, the options are clustered based on spot/strike price ratio.

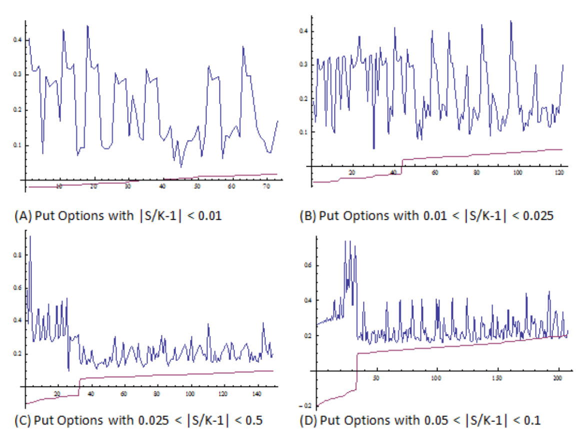

Figure 4.1 XEO put options grouped by spot/strike price ratio

Comparing the outcomes of put options implied volatility through the four different groups, as displayed in Figure 4.1, arranged based on the ratio of spot price over strike price, it can be seen that the standard deviation of the four groups does not show big difference. It might be worth mentioning that the minimal standard deviation is for the ratio ranging between 1.01 and 1.025.

On the other hand the mean value of the standard deviation of the implied volatility values has an ascending order. In parallel, as the ratio between Spot price and Strike price increases so does the mean value of Implied Volatility.

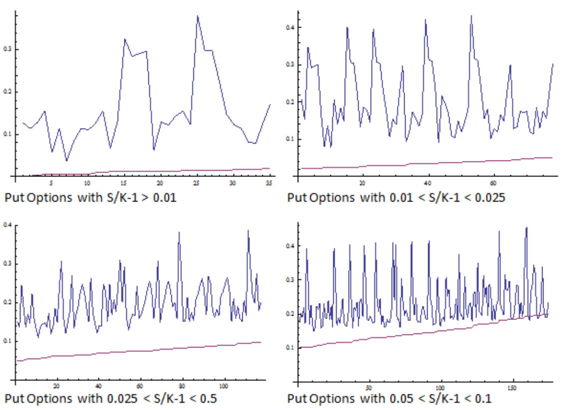

Figure 4.2 XEO Put options grouped by ratio, having S/K >1

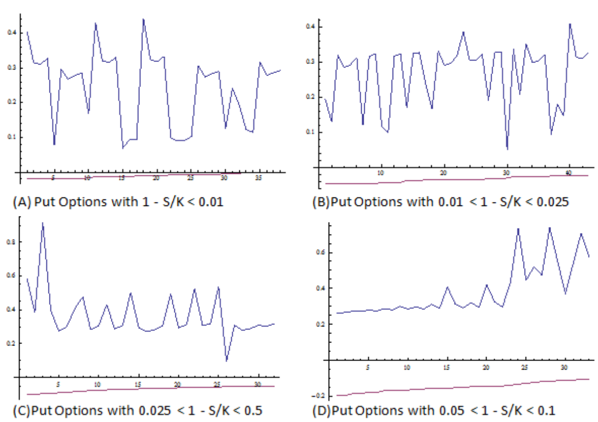

Figure 4.3 XEO Put options grouped by ratio, having S/K <1

Analyzing the put options (Figure 4.2), having the spot price greater than the strike price, implied volatility mean value is the lowest as the ratio is closer to 1 and is greatest at ratio ranging between 5% and 10%. On the other hand there is no significant difference in the mean value of implied volatility between the two groups of ratio ranging between 1 and 2.5 percent versus ratio ranging between 2.5 and 5 percent.

Simultaneously, an increase in the ratio of Spot versus Strike price shows positive impact on stability of implied volatility through lower values of standard deviation of the group implied volatility.

Whereas looking at the put options having strike price greater than the spot price, the implied volatility mean value and the ratio move in the same direction. As the price ratio increases from 1% to 2.5% to 5% up to 10%, the volatility does respectively from 24.5% to 26.6% to 36.6% up to 38.6%. Looking at the stability of those implied volatility values, the highest is where spot-strike price ratio ranges between 1 and 2.5 percent, followed by less than 1%, and the rest not showing difference.

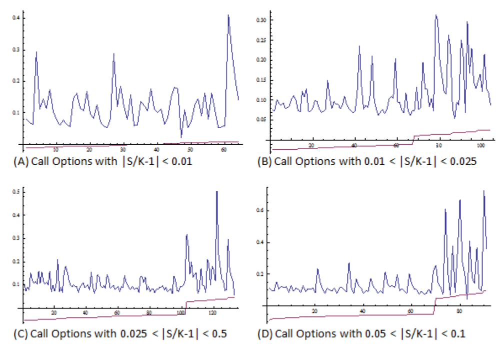

Figure 4.4 XEO Call options grouped by ratio

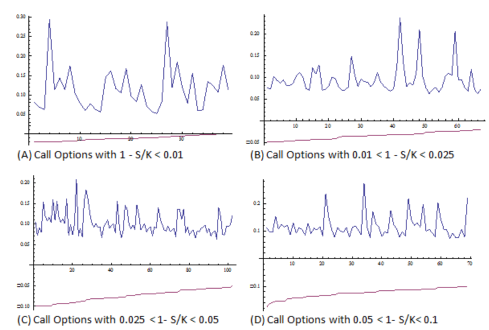

Figure 4.5 XEO Call options grouped by ratio, with S/K<1

Figure 4.4 shows the similar distribution as the Figure 4.1 does for the put options. Just in the case of the call options, the more stable implied volatility values correspond to options having the spot price being less than the strike price (S/K < 1).

Just as in the put options, an increase in the spot / strike price ratio within call options corresponds with lower values of standard deviation of the group implied volatility, again focusing on the out-of-money options.

5. CONCLUSIONS

Time to maturity has shown a negative correlation with implied volatility, in both call and put options. As such, the highest implied volatility values correspond for options having time to maturity below one month.

While looking for the stability of the implied volatility based on time to maturity, regardless of the moneyness value, the lowest standard deviation occurred for options having time to maturity ranging between three and six months.

While the characteristic of the moneyness effect on volatility, is that in both call and put options the more stable (having lover volatility standard deviation) implied volatility values corresponds to options' being more out-of-money.

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