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Decision Making Under the Risk Using Assets Liability Model (ALM): Case Study on Four Assets with no Transaction Costs

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Abstract

This paper focuses on the *Asset Liability Model (ALM)* with multistage stochastic model. The model is based on four assets with no transaction costs. The initial wealth is *Wo* should be invested wisely to meet the liability *L* at the end of the planned horizon *H*. the best possible decision is to keep the final wealth larger than liability L. Using excel solver we try to optimize solution as best possible decision that will at least meeting the liability L at the end of the terminal wealth.
The optimization case was adopted from the book (Brandimarte, 2011, pp.754-758) which has been modified with four assets and with no transaction costs. In adapting the case, we added additional returns for extra 2 stocks, while returns for the initial stock and bond remain.

1. INTRODUCTION

The main purpose of the stochastic programming is to find a wise and optimal solution when giving the managerial decision with uncertain data which are random not deterministic. And, programming in this terminology is related to the truth that some problems of certain cases could be mathematically modeled as liner or non-linear programming (Birge and Louveaux, 2011, pp.11.). The stochastic ALM model can be used in many different business decision makings. For instance it is proved as a very effective tool in in international capital market where managerial advices are necessarily needed (N. Topaloglou, et.al. 2008). This ALM model covers a way to manage assets to attain a certain and appropriate return and at the same time "maintaining a comfortable surplus of assets over existing and future liabilities" (Gülpinar, Pachamanova, 2013).

This paper is based on the case from the book (Brandimarte,2011.pp.754-758) using the multistage stochastic models is a simple asset- liability management (ALM) model and it the following text.

Introducing the Case of A multistage model: asset-liability management

The best way to introduce multistage stochastic models is a simple asset-liability management (ALM) model.[24] We have an initial wealth Wo, that should be properly invested in such a way to meet a liability *L* at the end of the planning horizon if. If possible, we would like to own a terminal wealth *WH* larger than *L;* however, we should account properly for risk aversion, since there could be some chance to end up with a terminal wealth that is not sufficient to pay for the liability, in which case we will have to borrow some money. A nonlinear, strictly concave utility function of the difference between the terminal wealth *WH,* which is a random variable, and the liability *L* would do the job, but this would lead to a nonlinear programming model.
As an alternative, we may build a piecewise linear utility function like the one illustrated in Fig. 13.10. The utility is zero when the terminal wealth *WH* matches the liability exactly. If the slope *r* penalizing the shortfall is larger than *q,* this function is concave (but not strictly).
The portfolio consists of a set of I assets. For simplicity, we assume that we may rebalance it only at a discrete set

of time instants t = 1, . . . , H 1, with no transaction cost; the initial portfolio is chosen at time t = 0, and the liability must be paid at time H. Time period t is the period between time instants t - 1 and t. In order to represent uncertainty, we may build a tree like that in Fig. 1. Each node nk in the tree corresponds to an event, where we should make some decision. We have an initial node no corresponding to time t = 0. Then, for each event node, we have two branches; each branch is labeled by a conditional probability of occurrence, P(nk I nz), where nz = a(nk) is the immediate predecessor of node nk. Here, we have two nodes at time t = 1 and

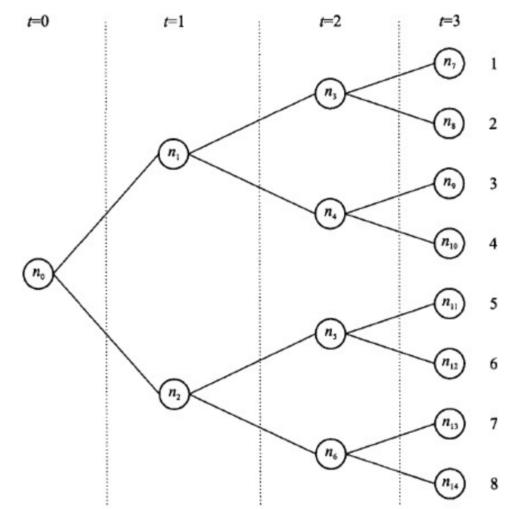

Fig. 13.11 Scenario tree for a simple asset–liability management problem.

Source: Adopted from P. Brandimarte,2011.pp.754-758

four at time t = 2, where we may rebalance our portfolio on the basis of the previous asset returns. Finally, in the 16 nodes corresponding to t = 3, the leaves of the tree, we just compare the terminal wealth with the liability and evaluate the utility function. Each node of the tree is associated with the set of asset returns during the corresponding time period. A scenario consists of an event sequence, i.e., a sequence of nodes in the tree, along with the associated asset returns. We have 8 scenarios in Fig. 13.11. For instance, scenario 2 consists of the node sequence (n0, *n1*, n3, n8). The probability of each scenario depends on the conditional probability of each node on its path. If each branch at each node is equiprobable, i.e., the conditional probabilities are always 2, each scenario in the figure has probability $\pi^s = \frac{1}{8}$, for $s = 1, \ldots, 8.$ fors = 1... 8. The branching factor may be arbitrary in principle; the more branches we use, the better our ability to model uncertainty; unfortunately, the number of nodes grows exponentially with the number of stages, as well as the computational effort.

At each node in the tree, we must make a set of decisions. In practice, we are interested in the decisions that must be implemented here and now, i.e., those corresponding to the first node of the tree; the other (recourse) decision variables are instrumental to the aim of devising a robust plan, but they are not implemented in practice, as the multistage model is solved on a rolling-horizon basis. This suggests that, in order to model the uncertainty as accurately as possible with a limited computational effort, a possible idea is to branch many paths from the initial node, and less from the subsequent nodes. Each decision at each stage may depend on the information gathered so far, but not on the future; this requirement is called a non-anticipativity condition. Essentially, this means that decisions made at time t must be the same for scenarios that cannot be distinguished at time t.To build a model ensuring that the decision process makes sense, there are two choices:

• We can introduce a set of decision variables x' , representing wealth allocated to asset i at time t on scenario s; we should force decision variables to take the same value when appropriate, by writing explicit nonanticipativity constraints for scenarios that cannot be distinguished at time t.

• We can associate decision variables with nodes in the scenario trees and write the model in a way that relates each node to its predecessors. We will illustrate the second alternative in detail, using the following numerical data:

• The initial wealth is 55.

• The target liability is 80.

• There are two assets, say, stocks and bonds; hence, I = 2.

• In the scenario tree of Fig. 13.11 we have up- and downbranches; in the (lucky) upbranches, total return is 1.25 for stocks and 1.14 for bonds; in the (bad) downbranches, total return is 1.06 for stocks and 1.12 for bonds. We see that bonds play the role of safer assets here. We also see that returns are a sequence of i.i.d. random variables, but more realistic scenarios can be defined.

• The reward rate q for excess wealth above the target liability is 1.

• The penalty rate r for the shortfall below the target liability is 4. Let us introduce the following notation:

• N is the set of event nodes; in our case N= {no,n1,n2... n14}.

Optimization problems are used often in different disciplines such as mathematics, science, economics and others. Optimal or near optimal result related to certain objectives is goal of each researcher that deals with these kinds of problems. In most cases, these problems are multi step constructed, so we execute several processes in a row rather than one. The aim of this paper is to analyze one case study of multistage stochastic problem using solver excel. This problem is about portfolio investment which represents periodical investments and sales of assets. Beside maximization of the wealth, this case has to satisfy some other constrains.

Each node n ∈ V, apart from the root node no, has a unique direct predecessor node, denoted by *a(n):* for instance, a(n3) = n1

• There is a set S ⊂ N of leaf (terminal) nodes; in our case *S*={n7, . . . , n14} ;for each node *s* ⊂ *S* we have surplus and shortfall variables *w*+ and *wL* ,related to the difference between terminal wealth and liability.

• There is a set *T* ⊂ *N* of intermediate nodes, where portfolio rebalancing may occur after the initial allocation in node no; in our case *T* = { n1 , . . . , n 6}, for each node n ∈{n0} U T there is a decision variable x in, expressing the money invested in asset *i* at node n. With this notation, the model may be written as follows:

$$
\begin{aligned}
\max \quad & \sum_{s\in\mathcal{S}} \pi^s (q w_+^s - r w_-^s) \\
\text{s.t.} \quad & \sum_{i=1}^{I} x_{i,n_0} = W_0 \\
& \sum_{i=1}^{I} R_{i,n} x_{i,a(n)} = \sum_{i=1}^{I} x_{in}, && \forall n \in \mathcal{T} \\
& \sum_{i=1}^{I} R_{is} x_{i,a(s)} = L + w_+^s - w_-^s, && \forall s \in \mathcal{S} \\
& x_{in}, w_+^s, w_-^s \geq 0
\end{aligned}
$$

where -R¿in is the total return for asset *i* during the period that *leads to* node n, and π^s is the probability of reaching the terminal node *s* ∈ *S;* this probability is the product of all the conditional probabilities on the path that leads from root node n0 to leaf node *s.* This is an LP model that may be easily solved by the simplex algorithm, resulting in the solution of Table 13.2. We may notice that in the last period the portfolio is not diversified, since the whole wealth is allocated to one asset, and we should wonder if this makes sense. Actually, it is a consequence of two features of this toy model:

• We are approximating a nonlinear utility function by a piecewise linear function, and this may imply "local" risk neutrality, so that we only care about expected return; we should use either a nonlinear programming model or a more accurate representation of utility with more linear pieces.

Table 13.2 Investment strategy for a simple ALM problem.

Node	Stocks	Bonds
n_0	41.4793	13.5207
n_1	65.0946	2.16814
n_2	36.7432	22.368
n_3	83.8399	0
n_4	0	71.4286
n_5	0	71.4286
n_6	64	0

The scenario tree has a very low branching factor, and this does not represent uncertainty accurately. However, the portfolio allocation in the last time period is not necessarily a critical output of the model: the real stuff is the *initial* portfolio allocation. As we pointed out, the decision variables for future stages have the purpose of avoiding a myopic policy, but they are not meant to be implemented (adopted from P. Brandimarte, 2011, pp.754-758).

2. ADAPTED MODEL WITH FOUR ASSETS

We extend the original model to four assets, three stocks and one bond.

Initial Data

The following is initial data in the table 1 and 2.

Table 1: Initial data of wealth, liability, reward, penalty and probability and return of assets

		Type	Ru	Rd
W0	55,00			
L	80,00	Rs1	1,26	1,07
q	1,00	Rs2	1,29	1,05
r	4,00	Rs3	1,24	1,08
pi	0,13	Rb	1,15	1,12

Where:

W0 – initial wealth/investment
L-liability
q - reward
r - penalty
pi - probability
Rs1-stock1
Rs2 – stock2
Rs3 – stock3
Rb – bond

Objective Function and Decision Tree

Adapted case of four assets will be based on the following objective function and the only difference is adding new variables of the additional two stocks, and placing them properly in both the objective function and all the constraints.

Given the above four possible investments (shown in Table 1), and their returns, there is no possibility of achieving the 80$ in all eight possible final outcomes at t=3. Knowing that the objective function gives four times as high priority to reaching the 80$ limit than it does to going further above that limit, it would rather be expected to have the outcome with minimal number of final returns being below the stated limit.

With this notation, the model may be written as follows:

$$\max \quad \sum_{s \in S} \pi^s (q w_+^s - r w_-^s)$$

$$\text{s.t.} \quad \sum_{i=1}^{I} x_{i,n_0} = W_0$$

$$\sum_{i=1}^{I} R_{i,n} x_{i,a(n)} = \sum_{i=1}^{I} x_{in}, \qquad \forall n \in \mathcal{T}$$

$$\sum_{i=1}^{I} R_{is} x_{i,a(s)} = L + w_+^s - w_-^s, \qquad \forall s \in \mathcal{S}$$

$$x_{in}, w_+^s, w_-^s \geq 0$$

Figure 1: Objective function of the adapted model
Source: Adopted from P. Brandimarte, 2011, pp.754-758).

To simplify this objective function we can say that:
Maximize the final outcome giving 4 times higher priority in getting it to be above 80 as oppose to total sum (where more outcomes could be below 80) subject to:

1. In t=0, sum of investments is equal to initial investment (W0=55).
2. Sum of investments (s1, s2, s3 and B1) is equal to the return of the previous stage investment
3. Portfolio return at the final stage is equated liability and adjusted wealth (negative if it is below liability value, positive otherwise).

The portfolio tree is like the following:

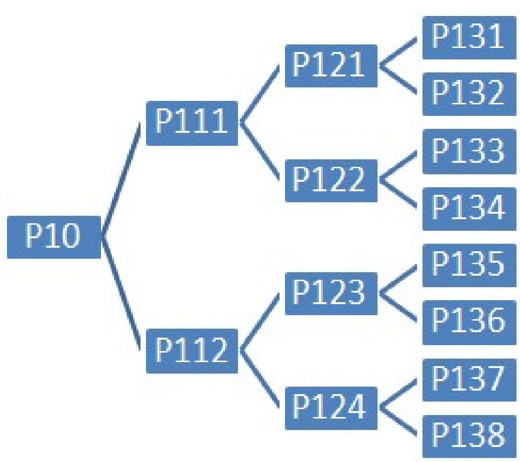

Figure 2: Decision tree for four assets

While, the initial portfolio assumption is:
P10=X10+X20+X30+X40

Where P represents the portfolio, while X10, X20 and X30 represent the three stocks and X40 does the single bond, all of them at initial time t=0.

Data Analysis and Interpretation

Once we set up our objective function with the portfolio decision tree and portfolio assumption we apply the objective function in the excel solver where we have:

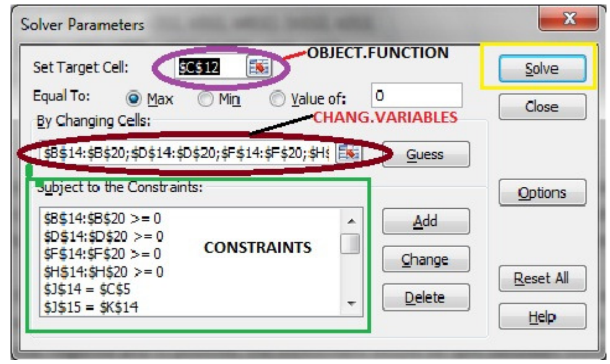

Figure 3: Excel solver application

After the excel solver is applied the results are as follows:

Table 3. Excel results of the objective function and portfolio assumption

	Max	2,061321					
Stock 1		Stock 2		Stock 3		Bond	
x10	55	x20	0	x30	0	x40	0
x111	65,45371	x211	3,846285	x311	0	x411	0
x112	6,593651	x212	0	x312	52,25635	x412	0
x121	0	x221	87,43339	x321	0	x421	0
x122	0	x222	0	x322	74,07407	x422	0
x123	0	x223	0	x323	46,96444	x423	26,14143
x124	63,49206	x224	0	x324	0	x424	0

The above results mean:

1. Based on the outcome, the initial (t=0) investment should consist of Stock 1 only, so initial investment of $55 will be invested in X10.
   At t=1, in case of positive outcome from t=0 the investment would be split between Stock 1 and Stock 2, $65.45 and $3.84 respectively. On the other hand, in case of negative outcome from first investment, the second investment would be spread between Stock1 and Stock 3, $6.59 and $52.26 respectively.
2. At t=2;
   If both t0 and t1 had positive outcomes, then the investment would be in Stock 3 only, the amount of $87.43 If t0 outcome was positive and t1 negative, then the investment would be in Stock 3 only. If t0 outcome was negative and t1 positive, the investment would be split between Stock 3 and Bond, $46.96 and $26.14 respectively. If both outcomes t0 and t1 were negative, then the t2 investment would be in Stock 1 only, the amount of $63.49
3. The t=3;
   The final outcome of those would be 112.79$, if $87.43 invested in Stock 3 gave positive return

The second possible final outcome of those would be 91.80$, if those same $87.43 invested in Stock 3 gave negative return.

The final outcomes of those would be 91.85$, if $74.07 previously invested in Stock 3 returned positively, while having negative return would makethe final outcome of 80$.

While assuming the portfolio of 46.96 invested in Stock 3 and 26.16 invested in Bond gave positive return, then the final outcomes of those would be 88.30$On the other hand, did that same portfolio respond negatively the final outcome of its would be 80$.

Were the return at t=1 and t=2 negative, the the final outcome of the portfolio would be 80$, if $63.49 invested in Stock 1 gave positive return. Otherwise, the final outcome of those would be 67.94$, if $63.49 invested in Stock 1 gave negative return

Table 4. Final outcomes for the stage t1,…t3.

Invested	x(n)*ru	x*rd	L+wu-wd
55	69,3	58,85	112,79
69,3	87,4333889	74,074075	91,81
58,85	73,105873	63,492063	91,85
87,43339	112,789071	91,805058	80,00
74,07407	91,8518519	80	88,30
73,10587	88,298554	80	80,00
63,49206	80	67,936508	80,00
Invested	Optimal final outcomes		67,94
at t-1	at the final stage t		

3. CONCLUSIONS

Application of the assets liability model (ALM) covers a very huge area of financial planning such as risk management, for individuals and institutions, government agencies banks and other financial institutions, pension plans, and insurance companies (Mulveya&Shettyb, 2004).

Problems and cases, or issues resolved in stochastic programming model “overcome the limitation of the static approaches” (Frauendorfer,Schurle, 2003).

Our model based on the case from the book (*P. Brandimarte, 2011, pp.754-758)*was tested with four assets with assumed rate of returns applying the excel solver techniques. Results show that only one of eight possible outcomes would be below the limit of 80$.

It would be interesting to further research how does one of the four (three stocks and one bond) dominate over others at each stage. As in majority of optimal outcomes investment is made in one of the four, while no single suggests spreading investment in three or four of them.

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