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Bulk queueing system with starting failure and single vacation

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ABSTRACT: To analyze an $M^{[X]}/G(a,b)/1$ queue with starting failure, repair and single vacation. This type of queueing model has a wide range of applications in production/manufacturing systems. Important performance measures and stability conditions are obtained. Further, we discuss some particular cases. Finally, numerical results of the proposed model have been derived.

Keywords:

General bulk service; Non-Markovian queue; Starting failure; Single vacation; Additional service.

1. INTRODUCTION

Manufacturing industries such as electronic industry and automobile, produce various types of goods to supply to customers. During the manufacturing process, it is impossible to guarantee that the production machine will not failure. Suppose the machine failed means the production will not complete on time. In such case the fast repair is required to complete the production on time.

Jinting Wang and Peng-Feng Zhou [2] analysed a feedback $M^{[X]}/G/1$ retrial queueing system with starting failures and admission control. Varalakshmi et al. [6] analysed a two phase service of M/G/1 retrial queueing system with immediate feedbacks, single vacation and starting failures. Rajadurai et. al [4] examined an M/G/1 retrial queue included orbit search, starting failures and single vacation. Thangaraj and Rajendran [5] examined single service and single vacation in bulk queueing system. Kempa [3] discussed the M/G/1/N-type finite capacity queueing system operating under single vacation policy. The model solved by embedded Markov chain and the total probability law a system of integral equations for the probability distribution of the length of the first loss series is built.

Vijayashree and Janani [7] considered an multi server Markovian queueing system subject to single exponential vacation. The stationary and transient probabilities for the number of customers during different server state are obtained explicitly for the system. Ayyappan and Karpagam [1] discussed non-Markovian bulk service queueing system with stand-by server and single vacation.

2. MATHEMATICAL DESCRIPTION

A bulk service queue with starting failure, repair, single is discussed. Server serves the clients under FCFS queue discipline and General Bulk Service Rule (GBSR). At that instant of service completion if the queue size is below ‘a’ then the server avails a single vacation otherwise, he starts regular service to the next batch of clients as per the GBSR.

On completion of the vacation period, if the queue size is minimum ‘a’ with no starting failure with probability ‘(1-p)’ the server starts a regular service otherwise, he immediately sent for repair. After repair completion the server starts the regular service. If the queue size '≤a ' on vacation completion, then the server remains idle and waiting for the next batch of arriving clients. Service, repair and vacation are assume to follow general(arbitrary) distribution.

3. PROBABILITIES AND NOTATIONS

λ - Arrival rate.

X- Group size random variable.

Pr(X=k)=g_k.

X(z) - the Probability Generating Function (PGF) of X.

S(.), R(.) and W(.) represent the Cumulative Distribution Function (CDF) of service time, repair and vacation time their corresponding probability density functions are s(w), r(w) and v(w) respectively.

$S^0(t)$, $R^0(t)$ and $W^0(t)$ represent the remaining time for service, remaining repair and vacation at time 't' respectively.

$\tilde{S}(\psi)$, $\tilde{W}(\psi)$ and $\tilde{R}(\psi)$ denotes the Laplace Stieltjes Transform (LST) of S, W and R respectively.

$I_n(t) \triangle t = \mathrm{Prob}\{N_1(t) = n, \phi(t) = 1\}, n \geq 0,$

$M_{r,j}(u,t) \triangle t = \mathrm{Prob}\{N_1(t) = r, N_2(t) = j, u \leq S^0(t) \leq u + \triangle t, \phi(t) = 2\},$

$W_n(u,t) \triangle t = \mathrm{Prob}\{N_2(t) = n, u \leq W^0(t) \leq u + \triangle t, \phi(t) = 3\}, n \geq 1,$

$R_n(u,t) \triangle t = \mathrm{Prob}\{N_2(t) = n, u \leq R^0(t) \leq u + \triangle t \phi(t) = 4\}, n \geq a.$

where $\phi(t) = 1, 2, 3$ and 4 represents server is in idle, busy, on vacation and repair respectively.

$N_1(t)$, $N_2(t)$ be no. of clients in service and queue at time 't' respectively.

4. DISTRIBUTION OF QUEUE SIZE

The Kolmogorov backward equation governing the system for the proposed model is:

$$\lambda I_0 = W_0(0) \quad (1)$$

$$\lambda I_n = W_n(0) + \sum_{k=1}^{n} I_{n-k}\lambda g_k, 1 \leq n \leq a-1 \quad (2)$$

$$-M'_{d,0}(u) = -\lambda M_{d,0}(u) + R_d(0)s(u) + (1-p)W_d(0)s(u) + (1-p)\sum_{k=0}^{a-1} I_k\lambda g_{d-k}s(u) + \sum_{i=a}^{b} M_{i,d}(0)s(u), a \leq d \leq b \quad (3)$$

$$-M'_{d,j}(u) = -\lambda M_{d,j}(u) + \sum_{k=1}^{j} M_{d,j-k}(u)\lambda g_k, j \geq 1, a \leq d \leq b-1 \quad (4)$$

$$-M'_{b,j}(u) = -\lambda M_{b,j}(u) + R_{b+j}(0)s(u) + (1-p)\sum_{k=0}^{a-1} I_k\lambda g_{b+j-k}s(u) + (1-p)W_{b+j}(0)s(u) + \sum_{i=a}^{b} M_{i,b+j}(0)s(u) + \sum_{k=1}^{j} M_{b,j-k}(u)\lambda g_k, \ j \geq b \quad (5)$$

$$-W'_0(u) = -\lambda W_0(u) + \sum_{i=a}^{b} M_{i,0}(0)v(u), \quad (6)$$

$$-W'_d(u) = -\lambda W_d(u) + \sum_{i=a}^{b} M_{i,d}(0)v(u) + \sum_{k=1}^{d} W_{d-k}(u)\lambda g_k, 1 \leq d \leq a-1 \quad (7)$$

$$-W'_d(u) = -\lambda W_d(u) + \sum_{k=1}^{d} W_{d-k}(u)\lambda g_k, \quad d \geq a \quad (8)$$

$$-R'_a(u) = -\lambda R_a(u) + pW_a(0)r(u) + p\sum_{k=0}^{a-1} \lambda I_k g_{a-k}r(u), \quad (9)$$

$$-R'_d(u) = -\lambda R_d(u) + pW_d(0)r(u) + p\sum_{k=0}^{a-1} \lambda I_k g_{d-k}r(u) + \sum_{k=1}^{d-a} R_{d-k}(u)\lambda g_k, \quad d > a. \quad (10)$$

While applying LST to the above equations (3) to (10), we get,

$$\psi\widetilde{M}_{d,0}(\psi) - M_{d,0}(0) = \lambda\widetilde{M}_{d,0}(\psi) - (1-p)W_d(0)\tilde{S}(\psi) - (1-p)\sum_{k=0}^{a-1} I_k\lambda g_{d-k}\tilde{S}(\psi) - \sum_{i=a}^{b} M_{i,d}(0)\tilde{S}(\psi)$$

$$-R_d(0)\tilde{S}(\psi), a \le d \le b \tag{13}$$

$$\psi\widetilde{M}_{d,j}(\psi) - M_{d,j}(0) = \lambda\widetilde{M}_{d,j}(\psi) - \sum_{k=1}^{j} M_{d,j-k}(\psi)\lambda g_k, j \ge 1, \quad a \le d \le b-1 \tag{14}$$

$$\psi\widetilde{M}_{b,j}(\psi) - M_{b,j}(0) = \lambda\widetilde{M}_{b,j}(\psi) - (1-p)\sum_{k=0}^{a-1} I_k\lambda g_{b+j-k}\tilde{S}(\psi) - (1-p)W_{b+j}(0)\tilde{S}(\psi) - \sum_{i=a}^{b} M_{i,b+j}(0)\tilde{S}(\psi) - R_{b+j}(0)\tilde{S}(\psi) - \sum_{k=1}^{j} M_{b,j-k}(\psi)\lambda g_k, j \ge 1 \tag{15}$$

$$\psi\widetilde{W}_0(\psi) - W_0(0) = \lambda\widetilde{W}_0(\psi) - \sum_{i=a}^{b} M_{i,0}(0)\widetilde{W}(\psi) \tag{16}$$

$$\psi\widetilde{W}_d(\psi) - W_d(0) = \lambda\widetilde{W}_d(\psi) - \sum_{i=a}^{b} M_{i,d}(0)\widetilde{W}(\psi) - \sum_{k=1}^{d} \widetilde{W}_{d-k}(\psi)\lambda g_k, \quad 1 \le d \le a-1 \tag{17}$$

$$\psi\widetilde{W}_d(\psi) - W_d(0) = \lambda\widetilde{W}_d(\psi) - \sum_{k=1}^{d} \widetilde{W}_{d-k}(\psi)\lambda g_k, d \ge a \tag{18}$$

$$\psi\tilde{R}_a(\psi) - R_a(0) = \lambda\tilde{R}_a(\psi) - pW_a(0)\tilde{R}(\psi) - p\sum_{k=0}^{a-1} \lambda I_k g_{a-k}\tilde{R}(\psi) \tag{19}$$

$$\psi\tilde{R}_d(\psi) - R_d(0) = \lambda\tilde{R}_d(\psi) - pW_d(0)\tilde{R}(\psi) - p\sum_{k=0}^{a-1} \lambda I_k g_{d-k}\tilde{R}(\psi) - \sum_{k=1}^{d-a} \tilde{R}_{d-k}(\psi)\lambda g_k, \quad d > a \tag{20}$$

Let us define the following PGF's:

$$\widetilde{M}_d(z,\psi) = \sum_{j=0}^{\infty} \widetilde{M}_{d,j}(\psi)\mathrm{z}^j, M_d(\mathrm{z},0) = \sum_{j=0}^{\infty} M_{d,j}(0)\mathrm{z}^d, a \le d \le b$$

$$\widetilde{W}(\mathrm{z},\psi) = \sum_{d=0}^{\infty} \widetilde{W}_d(\psi)\mathrm{z}^d, W(\mathrm{z},0) = \sum_{d=0}^{\infty} W_d(0)\mathrm{z}^d$$

$$\tilde{R}(\mathrm{z},\psi) = \sum_{d=a}^{\infty} \tilde{R}_d(\psi)\mathrm{z}^d, R(\mathrm{z},0) = \sum_{d=a}^{\infty} R_d(0)\mathrm{z}^d$$

5. PGF OF QUEUE SIZE

Theorem 1: If Ik, ck and mk are the steady state probabilities of 'k' clients in the queue, then the probability generating function of the queue length at an arbitrary epoch P(z) is

$$P(\mathrm{z}) = \frac{\begin{array}{l}(1-\tilde{S}(y(\mathrm{z})))\sum_{n=a}^{b-1}(\mathrm{z}^b-\mathrm{z}^n)(c_n+(1-p)\sum_{k=0}^{a-1} I_k\lambda g_{n-k}) \\ +(\mathrm{z}^b-1)[y(\mathrm{z})[(1-p)+p\tilde{R}(y(\mathrm{z}))]\sum_{k=0}^{a-1} I_k\mathrm{z}^k \\ +((1-\widetilde{W}(y(\mathrm{z})))+p\widetilde{W}(y(\mathrm{z}))(1-\tilde{R}(y(\mathrm{z}))))\sum_{k=0}^{a-1} m_k\mathrm{z}^k]\end{array}}{y(\mathrm{z})[\mathrm{z}^b-\tilde{S}(y(\mathrm{z}))]} \tag{21}$$

PGF of queue size at various completion epoch is

$$P_s(\mathrm{z}) = \frac{\begin{array}{l}(1-\tilde{S}(y(\mathrm{z})))[\sum_{r=a}^{b-1}(\mathrm{z}^b-\mathrm{z}^r)(c_r+(1-p)\sum_{k=0}^{a-1} I_k\lambda g_{r-k}) \\ +[((1-p)\widetilde{W}(y(\mathrm{z}))-1)+p\tilde{R}(y(\mathrm{z}))\widetilde{W}(y(\mathrm{z}))]\sum_{r=0}^{a-1} m_r\mathrm{z}^r \\ -y(\mathrm{z})I(\mathrm{z})[p(\tilde{R}(y(\mathrm{z}))-1)+1]]\end{array}}{y(\mathrm{z})[\mathrm{z}^b-\tilde{S}(y(\mathrm{z}))])} \tag{22}$$

The PGF of vacation completion epoch $\mathrm{P_v(z)}$ is

$$P_v(\mathrm{z}) = \frac{(1-\widetilde{W}(y(\mathrm{z})))\sum_{r=0}^{a-1} m_r\mathrm{z}^r}{y(\mathrm{z})} \tag{23}$$

The PGF of repair completion epoch $\mathrm{P_r(z)}$ is

$$P_r(\mathrm{z}) = \frac{p(1-\tilde{R}(y(\mathrm{z})))[\widetilde{W}(y(\mathrm{z}))\sum_{r=0}^{a-1} m_r\mathrm{z}^r - y(\mathrm{z})I(\mathrm{z})]}{y(\mathrm{z})} \tag{24}$$

6. IMPORTANT PERFORMANCE MEASURES

Probability of various state of the server.

Server is busy

$$P(B) = \frac{N_s{'''}D_s{''} - N_s{''}D_s{'''}}{3Dr{''}^2} \tag{25}$$

where

$$N_s{''} = -2(S_1)[\sum_{i=a}^{b-1}(b-i)(c_i + (1-p)\sum_{k=0}^{a-1} I_k\lambda g_{i-k}) + \lambda X_1 I(1) + \sum_{n=0}^{a-1} m_n[(1-p)W_1 + p(R_1 + W_1)]]$$

$$N_s''' = -3(S_2)[\sum_{i=a}^{b-1} (b-i)(c_i + (1-p)\sum_{k=0}^{a-1} I_k \lambda g_{i-k}) + \lambda X_1 I(1)$$

$$+(\sum_{n=0}^{a-1} m_n[(1-p)W_1 + p(R_1 + W_1)])]$$

$$+S_1[\sum_{i=a}^{b-1} (b(b-1) - i(i-1))(c_i + (1-p)\sum_{k=0}^{a-1} I_k \lambda g_{i-k})$$

$$+[\lambda X_2 I(1) + 2\lambda X_1 I'(1)] + 2pR_1 \lambda X_1 I(1)$$

$$+(\sum_{n=0}^{a-1} m_n[(1-p)W_2 + p(R_2 + 2R_1W_1 + W_2)]$$

$$+2\sum_{n=0}^{a-1} n m_n[(1-p)W_1 + p(R_1 + W_1)])]]$$

$$D_s'' = -2\lambda X_1[(b - S_1)]$$

$$D_s''' = -3\lambda X_2[(b - S_1)] - \lambda X_1[(b(b-1) - S_2)]$$

Server is on vacation

$$P(W) = \frac{[\lambda X_1[W_2 \sum_{n=0}^{a-1} m_n + 2W_1 \sum_{n=0}^{a-1} n m_n] - \lambda X_2 W_1 \sum_{n=0}^{a-1} m_n]}{2(\lambda X_1)^2} \quad (33)$$

Server is on repair

$$P(R) = \frac{p\lambda[X_1[R_2 \sum_{n=0}^{a-1} m_n + R_1((W_1 \sum_{n=0}^{a-1} m_n + \sum_{n=0}^{a-1} n m_n) + \lambda X_1 I'(1))] - X_2 R_1 \sum_{n=0}^{a-1} m_n]}{2(\lambda X_1)^2} \quad (34)$$

5.2. Expected Queue Length

The mean queue length E(Q) at an arbitrary time epoch is given by

$$E(Q) = \frac{Nr'''Dr'' - Nr''Dr'''}{3Dr''Dr''} \quad (35)$$

where

$$Nr'' = 2T_{11}\sum_{i=a}^{b-1} (b-i)d_i + T_{22}\sum_{k=0}^{a-1} I_k + \sum_{k=0}^{a-1} m_k$$

$$Nr''' = 3(T_{12}\sum_{i=a}^{b-1} (b-i)d_i + T_{11}\sum_{i=a}^{b-1} (b(b-1) - i(i-1))d_i)$$

$$+T_{23}\sum_{k=0}^{a-1} I_k + 3T_{22}\sum_{k=0}^{a-1} kI_k + T_{33}\sum_{k=0}^{a-1} m_k + 3T_{32}\sum_{k=0}^{a-1} km_k$$

$$Dr'' = 2\lambda X_1 Y_1$$

$$Dr''' = 3(\lambda X_2 Y_1 + \lambda X_1 Y_2)$$

$$T_{11} = -S_1$$

$$T_{12} = -S_2$$

$$T_{22} = -2b\lambda X_1$$

$$T_{23} = -3(b(b-1)\lambda X_1 + b\lambda X_2) + 2b\lambda X_1 p R_1$$

$$T_{32} = -2b(W_1 + pR_1)$$

$$T_{33} = 3(b(b-1) - b(W_2 + p(W_1R_1 + R_2)))$$

$$Y_1 = -(b - S_1)$$

$$Y_2 = (b(b-1) - S_2)$$

5.3 Expected Waiting Time

The expected waiting time is obtained by using the Little's formula as;

$$E(W) = \frac{E(Q)}{\lambda E(X)}$$

7. NUMERICAL EXAMPLES

This section offers with the numerical illustration of the proposed queueing model through variations within the parameters by using MATLAB software. We consider the following assumptions:

Batch size distribution of the arrival is geometric with mean 2. Let a=3 andn b=8.

Service time distribution is 2-Erlang.

Vacation and repair time is exponential with parameters $\Omega = 2$ and $\psi = 8$ respectively.

For various arrival rate and service rate the preformance measures E(Q) and E(W) are calculated.

Table 1

λ	**EQ**	**EW**
9	5.15	0.2861
10	6.122	0.3061
11	7.107	0.3231
12	8.105	0.3377
13	9.115	0.3506
14	10.14	0.3621
15	11.18	0.3727

Table 2

µ1	**rho**	**EQ**	**EW**
10.00	0.1000	1.5110	0.1511

10.25	0.0976	1.5078	0.1508
10.50	0.0952	1.5048	0.1505
10.75	0.0930	1.5021	0.1502
11.00	0.0909	1.4997	0.1500
11.25	0.0889	1.4974	0.1497
11.50	0.0870	1.4954	0.1495
11.75	0.0851	1.4936	0.1494
12.00	0.0833	1.4919	0.1492
12.25	0.0816	1.4903	0.1490
12.50	0.0800	1.4890	0.1489
12.75	0.0784	1.4877	0.1488
13.00	0.0769	1.4866	0.1487
13.25	0.0755	1.4855	0.1486
13.50	0.0741	1.4845	0.1485
13.75	0.0727	1.4836	0.1484
14.00	0.0714	1.4827	0.1483
14.25	0.0702	1.4820	0.1482
14.50	0.0690	1.4814	0.1481
14.75	0.0678	1.4807	0.1481
15.00	0.0667	1.4802	0.1480

8. CONCLUSION

In this article, we analysed a bulk service queue with starting failure, repair and single vacation. From the numerical results, it is observed that the performance measures E(Q) and E(W) are decreases as service rate and repair rate increase.

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